Micro III - June 2017 (Solution Guide)

1. Consider the following game G, where Player 1 chooses the row and Player 2 simultaneously chooses the column.

		Player 2		
		D	E	F
	A	$7, 7$	$0, 1$	$0, 3$
Player 1	B	$2, 2$	$3, 3$	$0, 0$
	C	$8, -1$	$2, 0$	$1, 1$

   (a) Show which strategies in G are eliminated by following the procedure of 'Iterated Elimination of Strictly Dominated Strategies'.

   SOLUTION: A is strictly dominated by C for Player 1, and can therefore be eliminated. After eliminating A, D is strictly dominated by E for Player 2, and can therefore be eliminated. No other strategy is strictly dominated for either player.

   (b) Find all Nash equilibria (NE), pure and mixed, in G. Show which NE gives the highest payoff to *both* players, and denote this equilibrium strategy profile by $e(1)$.

   SOLUTION: There are two NE in pure strategies: (B, E) and (C, F). There is also a mixed strategy NE, where Player 1 plays B with probability $1/4$ and C with probability $3/4$, and where Player 2 plays E with probability $1/2$ and F with probability $1/2$. The NE (B, E) gives the highest payoff to both players. Hence, $e(1) = (B, E)$.

   (c) Now consider the game $G(2)$, which consists of the stage game G repeated two times. Assume that players discount period-2 payoffs with factor $\delta \geq 1/2$. Define the average payoff of player $i \in \{1, 2\}$ in $G(2)$ as $(\pi_{i,1} + \pi_{i,2})/(1 + \delta)$, where $\pi_{i,t}$ refers to player i's payoff in period t.

   Find one pure strategy Subgame Perfect Nash Equilibrium (SPNE) where *both* players earn an average payoff that is strictly higher than their payoff in $e(1)$. (NOTE: make sure to consider deviations in any subgame). Denote the equilibrium strategy profile you found by $e(2)$.

   SOLUTION: Consider the following strategy s_1 for Player 1: 'Play A in period 1. Play B in period 2 if the period-1 outcomes was (A, D), and otherwise play C.' Consider the following strategy s_2 for Player 2: 'Play D in period 1. Play E in period 2 if the period-1 outcome was (A, D), and otherwise play F.' The strategy profile (s_1, s_2) implies NE play in period 2, in every subgame, so no deviation in period 2 can be profitable. To establish that (s_1, s_2) is a SPNE, it remains to show that no player has an incentive to deviate in period 1. Both players earn $7 + 3\delta$ on the equilibrium path. Player 2's payoff from deviating in period 1 can be no higher than $3 + \delta$, so such a deviation cannot be profitable. Player 1's payoff from deviating in period 1 can be no higher than $8 + \delta$, and such a deviation cannot be profitable when $\delta \geq 1/2$. Hence, (s_1, s_2) is a SPNE. Finally, since $(7 + 3\delta)/(1 + \delta) > 3$, both players earn a strictly higher average payoff than in $e(1)$. It follows that we can write $e(2) = (s_1, s_2)$.

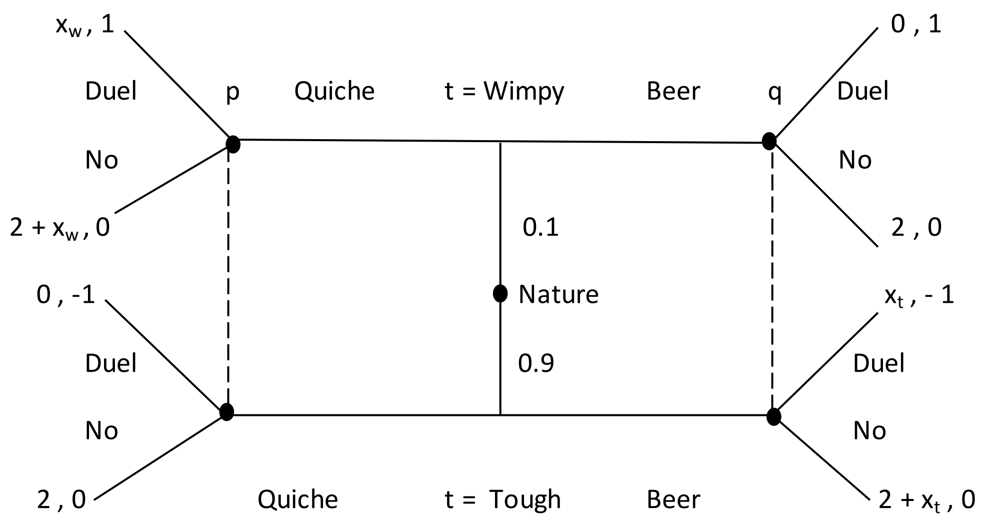
   (d) Now consider the game $G(\infty)$, which consists of the stage game G repeated infinitely many times. Continue to assume that players discount future payoffs with factor

$\delta \geq 1/2$. Define the average payoff of player $i \in \{1,2\}$ as $(\sum_{t=1}^{\infty} \delta^{t-1}\pi_{i,t})(1-\delta)$, where $\pi_{i,t}$ refers to player i's payoff in period t.

Find one pure strategy SPNE where *both* players earn an average payoff that is strictly higher than they earned in $e(2)$.

SOLUTION: Consider the following strategy $Trigger_1$ for Player 1: 'In period 1, play A. In any period $t \geq 2$, play A if (A, D) was the outcome of play in all periods $t' < t$; otherwise, play C.' Consider the following strategy $Trigger_2$ for Player 2: 'In period 1, play D. In any period $t \geq 2$, play D if (A, D) was the outcome of play in all periods $t' < t$; otherwise, play F.' The strategy profile $(Trigger_1, Trigger_2)$ implies NE play in every subgame after a deviation, so no deviation can be profitable off the equilibrium path. To rule out deviations on the equilibrium path, it is sufficient to consider both players' incentive to deviate in period 1. Both players earn $7/(1-\delta)$ on the equilibrium path. Player 2's payoff from deviating in period 1 can be no higher than $3+\delta/(1-\delta)$, so such a deviation cannot be profitable. Player 1's payoff from deviating in period 1 can be no higher than $8+\delta/(1-\delta)$, and such a deviation cannot be profitable when $\delta \geq 1/2$. Hence, $(Trigger_1, Trigger_2)$ is a SPNE. Since $7 > (7+3\delta)/(1+\delta)$, both players earn a strictly higher average payoff than in $e(2)$. It follows that we can write $e(\infty) = (Trigger_1, Trigger_2)$.

2. Consider the following signaling game. At each terminal node, the first number refers to the payoff of the Sender, and the second number refers to the payoff of the Receiver.



In words, the Sender must decide what to have for breakfast: quiche or beer. The Sender is either wimpy or tough. All else being equal, a wimpy type prefers quiche over beer, where $x_w > 0$ captures the intensity of this preference. Similarly, the tough type prefers beer over quiche, where $x_t > 0$ captures the intensity of this preference. The values of x_t and x_w are common knowledge. The Receiver observes what the Sender has for breakfast, and must then decide whether to challenge him to a duel. The Receiver only benefits from challenging (i.e. he wins the duel) if the Sender is wimpy. The Sender never benefits from being challenged, regardless of his type.

(a) Suppose for this subquestion that $x_w = 1$ and $x_t = 1$. Does a separating PBE exist in this game (yes or no)?

SOLUTION: No. In any separating equilibrium, the wimpy type will be challenged to a duel. He has a profitable deviation - to change his choice of breakfast and avoid being challenged.

(b) Explain for what values of $x_w > 0$ and $x_t > 0$ does a pooling PBE exist where both Sender types have beer, and find one such equilibrium. Explain for what values of $x_w > 0$ and $x_t > 0$ does a pooling PBE exist where both Sender types have quiche, and find one such equilibrium. Intuitively, why can the intensity of the Sender's preference over breakfast be important in a pooling equilibrium?

SOLUTION: First consider a PBE where the Sender pools on Beer, with beliefs $q = 0.1$ on the equilibrium path. It is straightforward to check that the Receiver's best response to Beer is No, given these beliefs. Thus, the tough type earns $2 + x_t > 2$ on the equilibrium path, and has no incentive to deviate. The wimpy type earns 2 on the equilibrium path. He has no incentive to deviate if and only if both (i) $x_w \leq 2$, and (ii) the Receiver responds to Quiche by playing Duel. This response is optimal for the Receiver if $p \geq 1/2$. Thus, the PBE is $(BeerBeer, DuelNo; p \geq 1/2, q = 0.1)$, whenever $0 < x_w \leq 2$ and $0 < x_t$. Now consider a PBE where the Sender pools on Quiche, with beliefs $p = 0.1$ on the equilibrium path. Just as above, the Receiver's optimal action is No on the equilibrium path. The wimpy type earns $2 + x_w > 2$ in equilibrium, and has no incentive to deviate. The tough type earns 2 on the equilibrium path. By the same logic as above, the PBE is $(QuicheQuiche, NoDuel; p = 0.1, q \geq 1/2)$, whenever $0 < x_w$ and $0 < x_t \leq 2$. Intuitively, in a pooling equilibrium, the Sender never faces a duel. But the Sender type who dislikes the breakfast he eats in equilibrium will want to deviate if the gains from eating his preferred breakfast outweigh the losses from being challenged to a duel. This means, for a pooling equilibrium to exist, the Sender's preference for one breakfast over another cannot be too strong.

(c) Now suppose again that $x_w = 1$ and $x_t = 1$. Using your answers in part (a) and (b), and referring to Signaling Requirements 5 and 6, what equilibrium do you think is the most likely to be played? What will the Sender have for breakfast? Will the Receiver to end up challenging the Sender to a duel?

SOLUTION: The answers in (a) and (b) identify two equilibria when $x_w = 1$ and $x_t = 1$: $(BeerBeer, DuelNo; p \geq 1/2, q = 0.1)$, and $(QuicheQuiche; NoDuel, p = 0.1, q \geq 1/2)$. Both equilibria satisfy Signaling Requirement 5, since no message is strictly dominated for either type. Pooling on Quiche does not satisfy Signaling Requirement 6, since Beer is equilibrium dominated for the weak type but not the tough type. Pooling on Beer satisfies Signaling Requirement 6 if $p = 1$, since Quiche is equilibrium dominated for the tough type but not the wimpy type. This suggests that pooling on Beer is the most reasonable equilibrium. Thus, the most likely outcome is that the Sender has beer for breakfast, regardless of his type, and the Receiver does not challenge him to a duel.

3. Two consumers are considering whether to buy a product that exhibits network effects. The payoff from buying depends on the choice of the other consumer. That is, for each consumer $i \in \{1, 2\}$, the payoff U_i from buying depends on three terms: the consumer's type, θ_i, which represents his intrinsic valuation of the product; a potential network payoff $\lambda > 0$, which consumer i only obtains if consumer $j \neq i$ also buys; and the price p. Specifically, buying yields $U_i = \theta_i + \lambda - p$ if consumer j also buys, or $U_i = \theta_i - p$ if

consumer j does not. Not buying the product gives a payoff of zero. Each consumer's type is drawn from a uniform distribution on $[0,1]$ and is private information. For all parts of this question, you can assume the following parameter values: $\lambda = 1/4$ and $p = 1/2$.

(a) Suppose consumers must simultaneously decide whether or not to buy, so the strategic situation they face can be seen as a static game of incomplete information. The Bayesian-Nash equilibrium of this game will be characterized by a threshold value of $\theta \in (0,1)$, which you can label as θ^*. What is the equilibrium probability that each consumer buys the product, in the Bayesian-Nash equilibrium of this game?

SOLUTION: The incentive to buy is increasing in type, so the equilibrium will have a threshold structure: type θ_i will buy if and only if $\theta_i \geq \theta^*$, for some cutoff θ^*. From the perspective of consumer i, the probability that consumer j buys is therefore $1-\theta^*$, so the expected payoff from buying is $\theta_i + \lambda(1-\theta^*) - p$. Consumer i will be indifferent about buying if $\theta_i = \theta^*$. This gives the condition $\theta^* + \frac{1}{4}(1-\theta^*) - \frac{1}{2} = 0$, or equivalently $\theta^* = 1/3$. The probability that each consumer buys is therefore $2/3$.

(b) Now consider the following modified situation. Consumer 1 is given the product for free. Consumer 2 knows this, and understands that his own payoff from buying is $U_2 = \theta_2 + \lambda - p$ for sure. Think of the strategic situation facing consumer 2 as a static game (with only one player). What is the equilibrium probability that consumer 2 buys the product, in the (Bayesian-Nash) equilibrium of this game? Briefly comment on any difference with your answer in part (a).

SOLUTION: Consumer 2 buys if and only if $\theta_2 + \frac{1}{4} - \frac{1}{2} \geq 0$, or equivalently $\theta_2 \geq \frac{1}{4}$. The probability that consumer 2 buys is therefore $3/4$. This probability is higher than in part (a). Intuitively, consumer 2 now knows for sure that he will enjoy a positive network payoff from buying the product, which increases his incentive to buy.

(c) One way to interpret part (a) is that the firm selling the product follows a 'standard' marketing approach, where it releases the product simultaneously to both consumers. One way to interpret part (b) is that the firm follows a 'seeding' marketing approach, giving away the product to one consumer for free, in the hopes of convincing the other consumer to buy. Given these interpretations, and using your answers in parts (a) and (b) to calculate firm revenues, argue whether a 'standard' or a 'seeding' approach is more profitable in this situation, and briefly explain why this is the case.

SOLUTION: Part (a) implies expected revenues of $2 * (2/3) = 4/3$. Part (b) implies expected revenues of $1 * (3/4) = 3/4$. Hence, in this particular situation, a standard approach is more profitable. By giving away the product to some consumers, a seeding approach reduces the size of the effective market (bad for revenue), but increases the probability that those consumers remaining in the market will buy (good for revenue). In this particular situation, the former effect dominates the latter, so that seeding is not profitable.